One-Dimensional Elastic Consolidation
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1 Description

In the oedometer for confined compression test, a sample of porous material saturated with fluid rests on a porous filter in an impervious chamber [Fig. 1]. At the start of the test a force load $F$ is applied in a very short time through an impervious piston above the sample. We assume that there is no friction between the chamber and the sample and that both the solid and the fluid components are nearly incompressible. Therefore, the sample will deform mainly by expelling fluid through the filter. At the filter side we consider a zero pressure constraint for the fluid. The solid material of the solid is assumed to be linear elastic and weightless, with a constant permeability. Initially the load will be balanced by the fluid pressure only, with a steep pressure gradient at the filter side. This gradient initiates fluid flow, thus allowing gradual deformation. The deformation leads to increasing effective stress and decreasing fluid pressure. Ultimately, the load will be balanced by the solid material only.

The analytical solution for the piston settlement by this one-dimensional consolidation was found by Terzaghi\textsuperscript{1}. In this example we perform a mixture analysis for a load $F$ causing a unit distributed force in a $1 \times 1$ m cylindrical sample with drained Young’s modulus $E_D = 1000$ Pa, Poisson’s ratio $\nu = 0$, modified permeability $k = 0.001$ m$^2$/(Pa.s), porosity $n = 0.55$ and compression moduli $K_s = 6666.67$ Pa for solid and $K_f = 100000$ Pa for fluid.

\textsuperscript{1}Terzaghi, \textit{Theoretial Soil Mechanics}, 1943
2 Finite Element Model

We start a new project for an axial symmetric structural analysis with mixture flow-stress option. We use quadratic mesh order. The model size is set to 10 m (-5 to 5 m) to include the entire model. We choose meter for the unit length, ton for mass and degree for angle. The units and the directions are displayed in the Reference system section of the Geometry browser [Fig. 3] [Fig. 4].

Main menu → File → New [Fig. 2]
Geometry browser → Reference system → Units [Fig. 3]
Property Panel [Fig. 4]

Figure 2: New project dialog
Figure 3: Geometry browser - units
Figure 4: Properties panel - units
2.1 Geometry

We create the axial symmetric sample with the name *Soil* by defining a line in $Y$ direction with length 1 m and extruding it in $X$ direction in 0.5 m.
2.2 Properties

We use regular solid ring elements and linear elastic material with parameters $E = 1000 \text{kN/m}^2$, $\nu = 0$, $\rho = 2 \text{T/m}^3$, $n = 0.55$. We include the mixture flow-stress aspect with the following parameters: $k = 0.001 \text{m}^3/\text{s}/\text{T}$ and the bulk-stiffness $K_s = 6666.67 \text{kN/m}^3$ and $K_f = 1e5 \text{kN/m}^3$. 

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2.3 Boundary Conditions

We define the following boundary conditions to the edges of the sample: 

i) support displacements in $X$ direction at the edge in the right-hand side, support displacements in $Y$ direction at the bottom edge and 

iii) support pore-pressure potentials at the bottom edge.

Main menu ➔ Geometry ➔ Assign ➔ Add supports ➔ [Fig. 12] – [Fig. 14]
We define one tying from the left top vertex as master and the top edge as slave for the vertical displacements.

Figure 16: Tyings at top edge

Figure 17: Master vertex

Figure 18: Slave edge
2.4 Loads

We define a nodal force in the vertical direction at the top left vertex, equivalent to a unit force applied to the top surface of the cylinder \( A = 0.5 \times 0.5 \times \pi \). We define a time curve for that load.

Figure 19: Attach load

Figure 20: Geometry view - load
We define a time curve for that load.

Figure 21: Geometry browser - loads

Figure 22: Edit time dependent factors
2.5 Mesh

We define a gradual mesh seeding from 0.01 m to 0.1 m at the axis of symmetry.

Figure 23: Input for gradual mesh seeding

Figure 24: Left edge for gradual mesh seeding
We set the number of divisions equal to 1 at the top and bottom edges.

![Figure 25: Input for unit mesh seeding]

![Figure 26: Top and bottom edges for unit mesh seeding]
We create the mesh.

Figure 27: Finite element mesh
3 Nonlinear Transient Analysis

3.1 Commands

We define a nonlinear analysis with name *Consolidation*. We activate the transient nonlinear effect with Euler backward scheme.

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**Figure 28**: Analysis browser

**Figure 29**: Nonlinear properties

**Figure 30**: Transient settings
We remove the default load steps execute block and define a new time steps execute block and rename this as *Time stepping*. We define the time-steps as: 0, 0.01, 0.015, 0.025, 0.05(38).

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**Analysis browser** → Nonlinear → Structural nonlinear → new execute block → Remove [Fig. 31]

**Analysis browser** → Nonlinear → Structural nonlinear → Add... → Execute steps - Time steps [Fig. 32]
For the output we choose the user selection mode for the following result items: total translational displacements, total pore-pressure potential degrees of freedom (displacements), effective Cauchy stresses, total Cauchy stresses and total pore-pressures. We run the analysis.
3.2 Results

We switch off the deformed shape representation. We define a probing curve along the right-hand edge, with the diagram oriented in the global X direction with a scale factor of 5 and 50 intervals between points.

For the contour plots, we set the color scale limits to specified values and define minimum value of -1 and maximum value of 0 for stress results and minimum value of 0 and maximum value of 1 for pore-pressure results. We can also change the bounding colors, if needed.
In the beginning, the sample is not loaded and thus pore-pressure and effective vertical stresses are zero.

Figure 38: Vertical effective stress at start

Figure 39: Pore pressure at start
Immediately after loading (time step 2 at time = 0.1 s) the fluid carries most of the load applied to the sample. The bottom edge where the fluid flows out we can observe a strong gradient until zero. This is the area where the effective stresses are increasing.

Figure 40: Vertical effective stress just after start

Figure 41: Pore pressure just after start
After 0.55 s (time step 5) the pore-pressure and effective stresses present similar values in the full domain.

![Vertical effective stress at 0.55 s](image1)

![Pore-pressure at 0.55 s](image2)

**Figure 42: Vertical effective stress at 0.55 s**

**Figure 43: Pore-pressure at 0.55 s**
After 19 s (time step 42) the pore-pressures are again reduced to almost zero in the entire sample and the force is carried by the effective stresses.
Figure 46 shows the graph of vertical displacement versus time for the right upper node. We can observe that within 2 s the sample compacts 1 mm, which matches its undrained loading condition.
Appendix A  Additional Information

Folder:  Tutorials/ElasticConsolidation

Number of elements \( \approx 14 \)

Keywords:
- ANALYS:  flowst mixtur nonlin physic transi.
- CONSTR:  suppor tying.
- ELEMEN:  axisym cq16a.
- LOAD:  force node time.
- MATERI:  elasti isotro permea porosi.
- OPTION:  backwa direct newton nonsym regula units.
- POST:  binary ndiana.
- PRE:  diana.
- RESULT:  cauchy displa effect pressu stress total.

References:

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