Post-tension Load in Diana

Outline

1 Introduction

The *prestressing* technique consists on the pre-loading of a structural member, such that the internal forces, generated by the pre-load, counteract the internal forces from the applied loads. *Post-tensioning* is one of the methods employed for prestressing of concrete structural members, wherein a tension load is applied to reinforcement tendons encased in ducts which are embedded inside the concrete. This tension is resisted by an anchorage assembly at the ends of the structural member, thereby translating it to a compressive load in the concrete. Later, depending on the design, the tendons are either bonded to the concrete, e.g. by grouting the ducts, or left unbonded. The application of post-tensioning and the associated losses can be modeled in DianaIE by applying the load type *Post-tensioning load* to embedded reinforcement.

In this tutorial, the features of the post-tensioning load are illustrated by means of a two-dimensional plane stress model of an unbonded tendon embedded inside of a continuous concrete beam. Since the focus of this tutorial is prestressing of concrete, only steps pertaining to the assignment of the prestressing load are presented in detail. Other steps related to creating the geometry and assigning other properties are not presented. For detailed information on *Post-tensioning load*, please see the Diana *Documentation*.

2 Finite Element Model

2.1 Geometry and Properties

The model presented in Figure 1 consists of two shapes: *Concrete* and *Tendon*. *Concrete* represents an end span of a continuous beam upon which we apply the prestressing load. The left end is the anchoring end where the beam is simply supported at the bottom. Symmetry conditions are considered on the right end. From left to right, the profile of the *Tendon* can be represented by two parabolas $g(X)$ and $h(X)$ [Fig. 1]:

$$g(X) = 0.006894 \times (X - 8.52)^2 + 0.2 \tag{1}$$

$$h(X) = -0.1 \times (X - 8.52)^2 + 2.296 \times (X - 8.52) - 12.13 \tag{2}$$

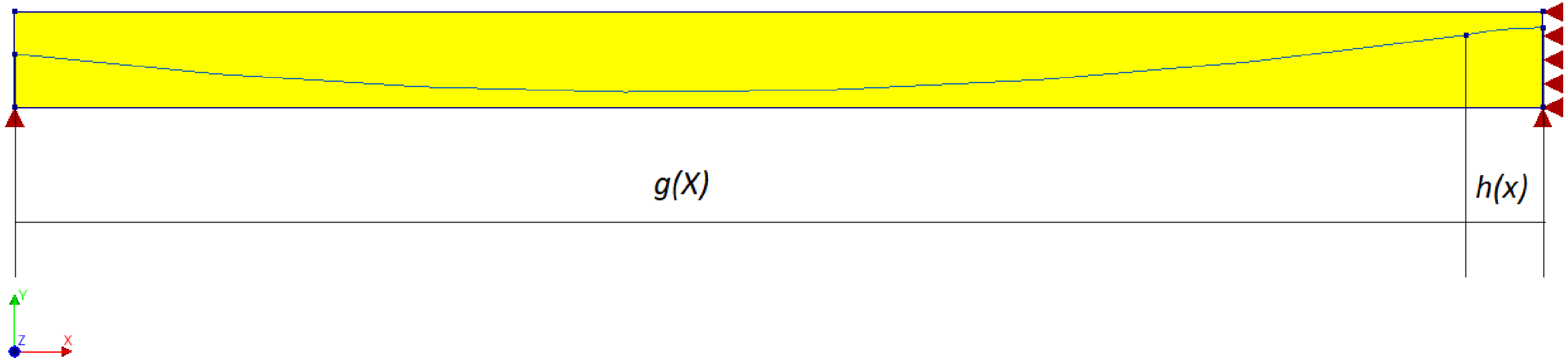

Figure 1: Model - geometry and supports

The coordinates of the shapes[1] are listed in Table 1.

Table 1: Coordinates of shapes

	X	Y
Concrete		
	0	0
	20	0
	20	1.25
	0	1.25
Tendon[1]		
	0	0.7
	8.5	0.2
	19	0.94904
	19.5	1.02404
	20	1.04904

In brief terms, the model has the following characteristics:

- properties:
  element class concrete: regular plane stress
  material model concrete: linear elastic isotropic (E = 30e+03 MPa, $\nu = 0.2$)
  element geometry concrete: thickness of 0.5 m
  material model tendon: linear elastic isotropic (E = 200e+03 MPa), reinforcement not bonded to mother element
  element geometry tendon: diameter 32 mm.
- supports:
  concrete left bottom and right bottom vertex: T2=0
  concrete right symmetry edge: T1=0
- mesh:
  mesh order: quadratic
  element size: 0.25 m

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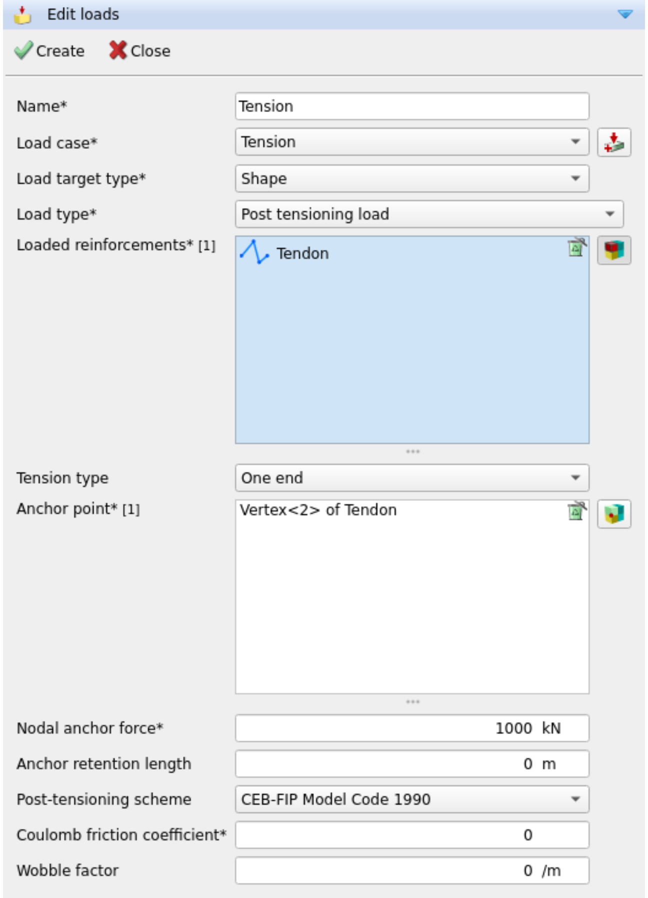
[1] For the tendon, two curves are created with three points each and then joined together

2.2 Post-tensioning Load

We define the *Post-tensioning load* [Fig. 2]. We specify the *Tendon* as a loaded reinforcement. Since we have considered symmetry in our model, we select tension type as *One end* and specify the leftmost vertex of the tendon as the anchoring point [Fig. 3]. Further, we specify a value of 1000 kN for the nodal anchor force. The remaining inputs correspond to the prestress losses, which we will consider in the following sections. At this point we keep them at zero, considering a situation without any loss of force along the length of the tendon.

DianaIE

Main menu → Geometry → Assign → Loads [Fig. 2]


Edit loads

Create Close

Name*: Tension
Load case*: Tension
Load target type*: Shape
Load type*: Post tensioning load
Loaded reinforcements* [1]: Tendon
Tension type: One end
Anchor point* [1]: Vertex<2> of Tendon
Nodal anchor force*: 1000 kN
Anchor retention length: 0 m
Post-tensioning scheme: CEB-FIP Model Code 1990
Coulomb friction coefficient*: 0
Wobble factor: 0 /m

Figure 2: Define post-tensioning load

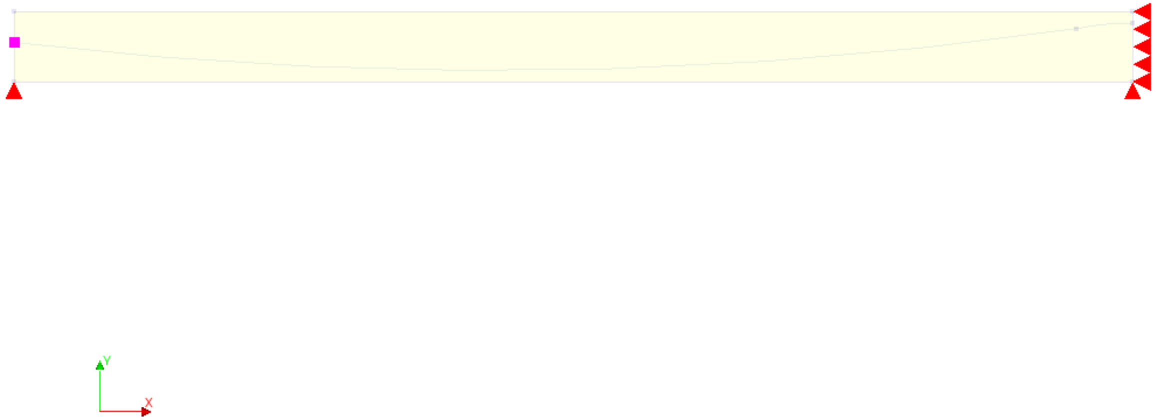


Figure 3: Anchor point of the tendon

2.3 Mesh

We generate the mesh [Fig. 4].

DianaIE

Main menu → Geometry → Mesh → Generate mesh [Fig. 4]

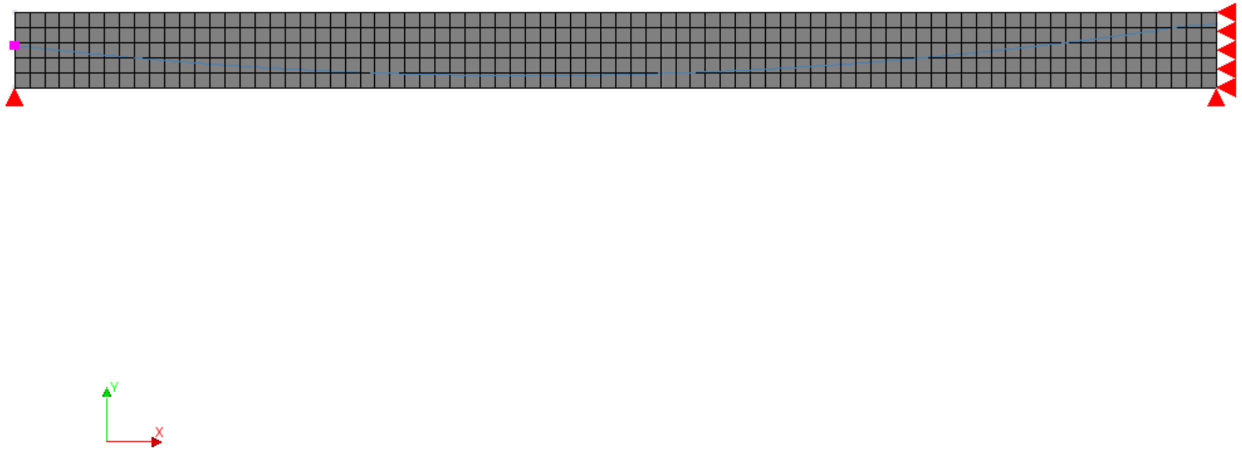


Figure 4: Finite element mesh

3 Analysis

We add a new analysis, rename it to *Linsta* and add a *Structural linear static* command to it. We run the analysis

DianaIE

Main menu → Analysis → Add analysis
Analysis browser → Analysis1 → Rename → Linsta
Analysis browser → Linsta → Add command → Structural linear static [Fig. 5]
Main menu → Analysis → Run analysis

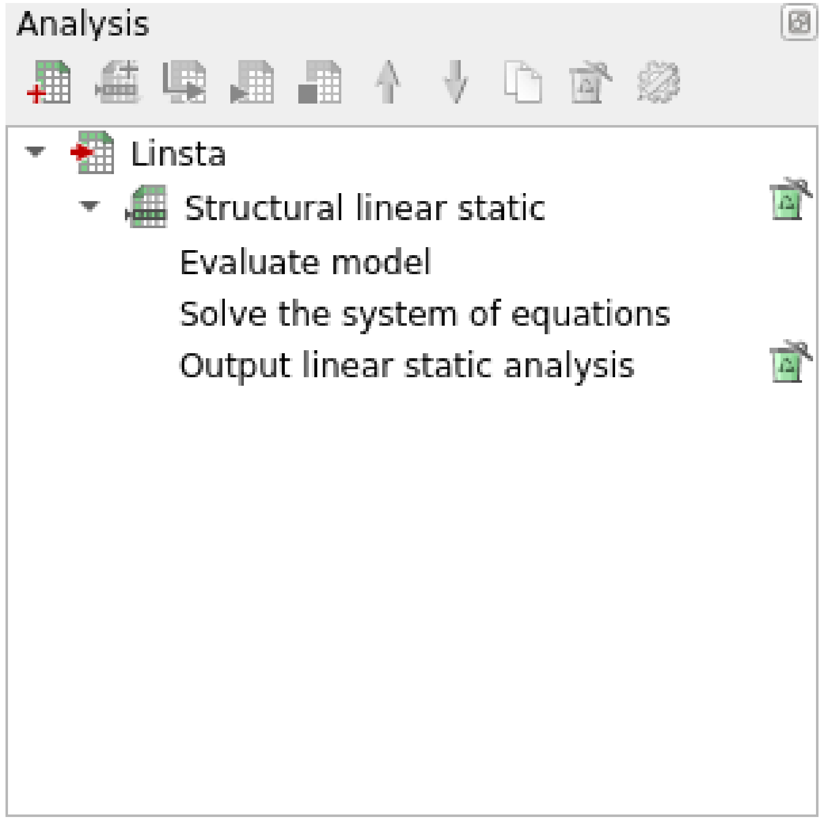


Figure 5: Analysis browser

3.1 No Prestress Loss

We check results for the load defined in the previous section. Since we did not considered any prestress losses, we refer to this case as *No Prestress Loss*. At the mid span of the beam, compressive stresses develop at the bottom [Fig. 7]. At the intermediate support compressive stresses develop at the top. These stresses will counteract the downward vertical load that will be applied on the structure.

DianaIE

Results browser → Output → Nodal results → Displacements → DtXY [Fig. 6]
Results browser → Output → Element results → Cauchy Total Stresses → SXX [Fig. 7]

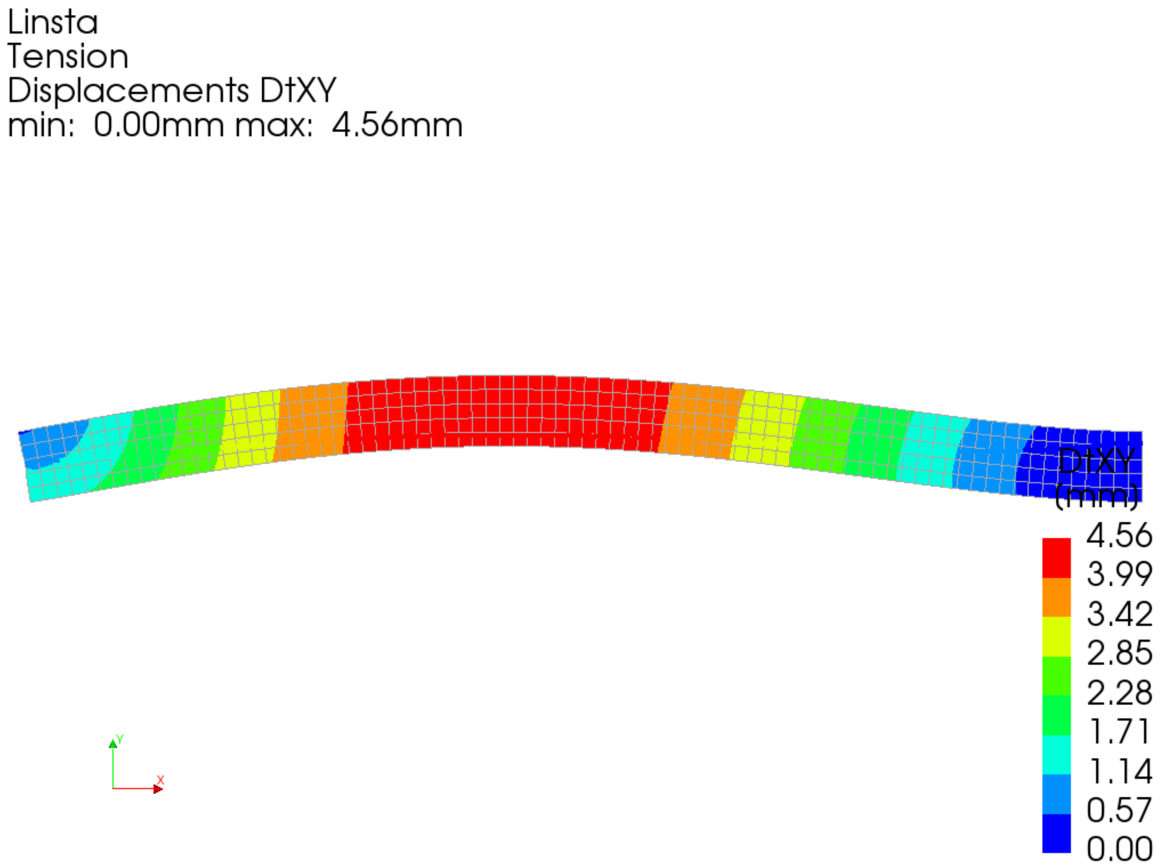


Figure 6: Total displacements in the concrete

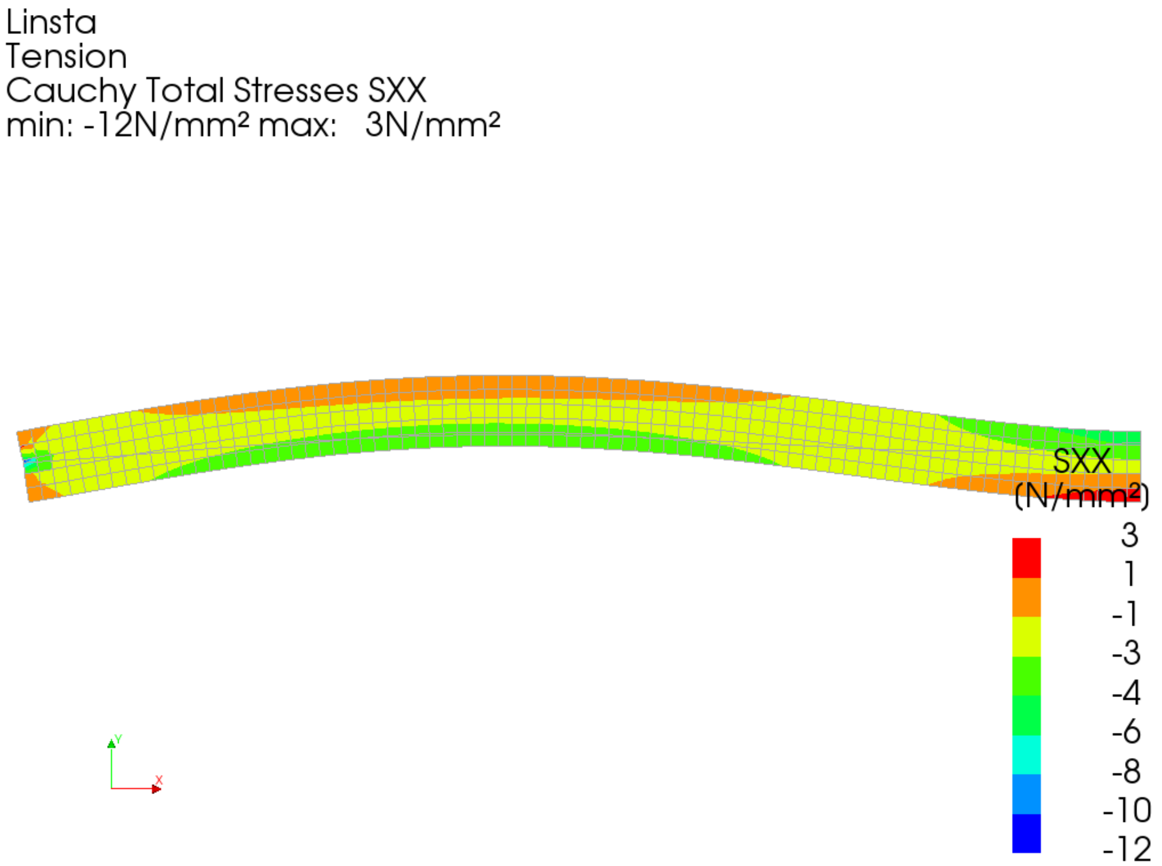


Figure 7: Longitudinal stresses in concrete

As expected a constant tensile force of 1000 kN is observed along thee length of the tendon [Fig. 9].

DianaIE

Property Panel → Results → Diagram plot settings → Scale factor → 10
Results browser → Output → Nodal results → Displacements → DtXY 🖱 → Show line diagram [Fig. 8]
Results browser → Output → Reinforcements results → Reinforcement Cross-section Forces → Nx [Fig. 9]

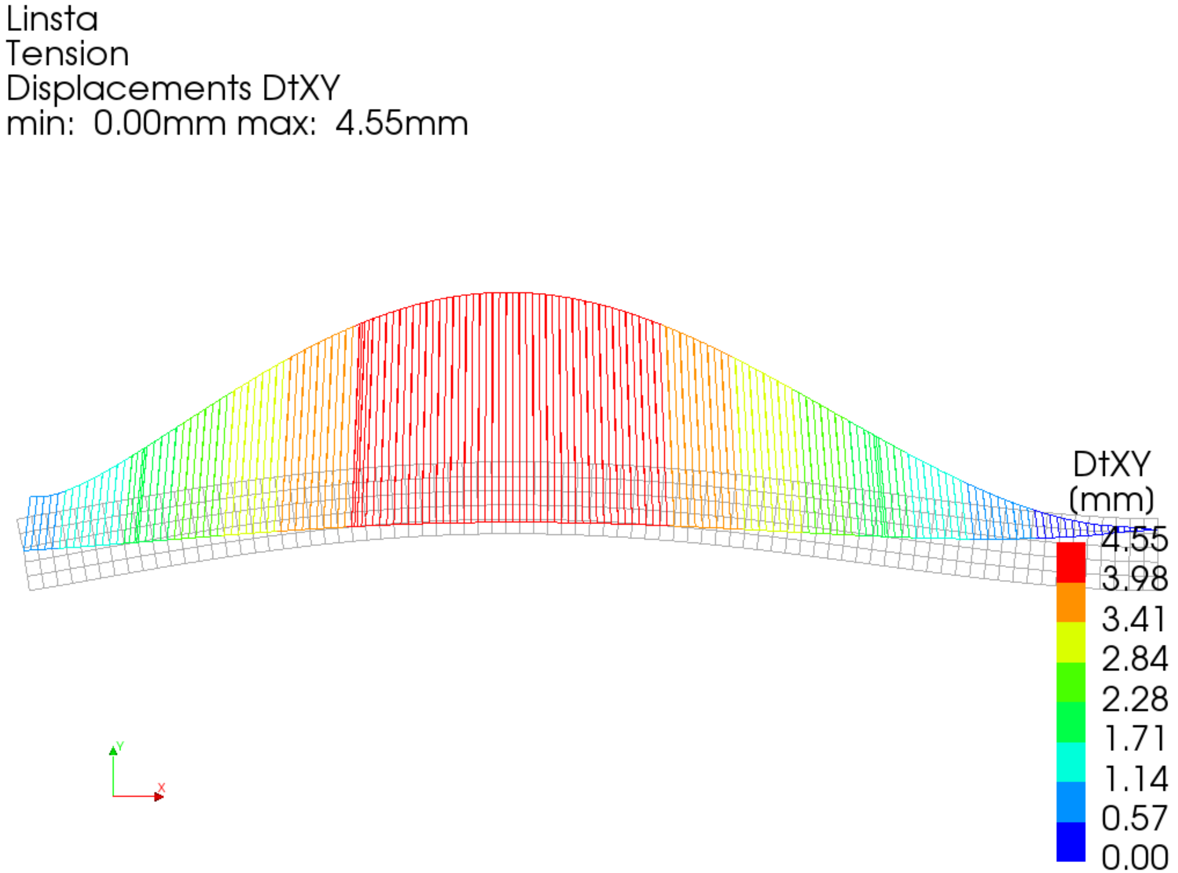


Figure 8: Displacements in the tendon

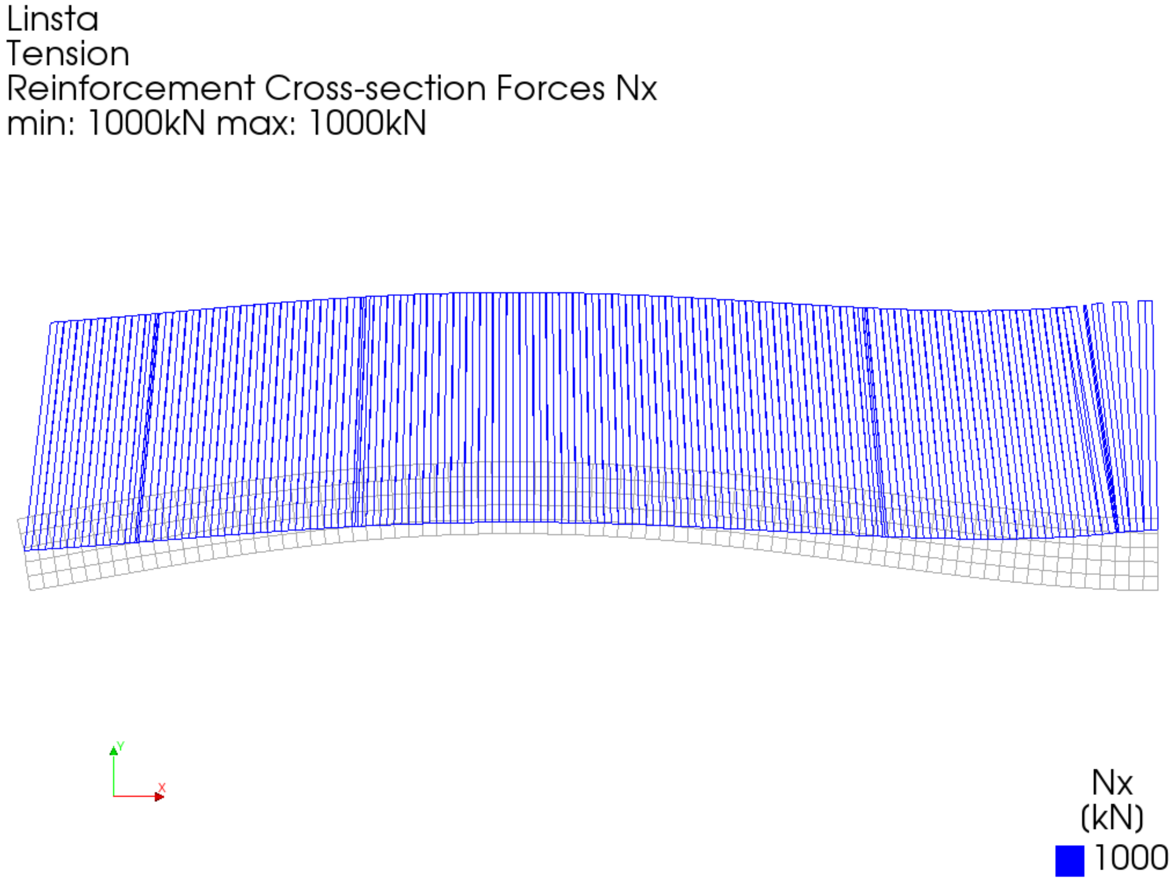


Figure 9: Cross-section force Nx along the tendon

3.2 Prestress Losses

Generally, the force in the tendon decreases with the increment of the distance from the anchorage device. This loss of force can be due to a variety of factors. Some prestress can be lost due to friction and penetration of the anchorage in the concrete. These two losses can be considered in the *Post tensioning load*. We first consider them individually and then study their combined effect.

3.2.1 Friction

Frictional forces are generated at the interface between the tendon and the surrounding duct [Fig. 10]. At this interface the ratio between the force per unit length along the axis of the tendon and the force per unit length normal to the axis of the tendon is given by the *Coulomb friction*. Starting from this, the expression for force at a given point in the tendon can be analytically expressed as

$$P = P_0 * e^{-\mu(\kappa+\phi_1)\Delta r} \tag{3}$$

where:

P is the force at a given point in the tendon

P_0 is the force at the reference point

μ is the Coulomb friction coefficient

κ is the curvature at a given point in the tendon

ϕ_1 is the Wobble parameter (extra curvature in order to account for local irregularities)

Δr is the distance from the reference point

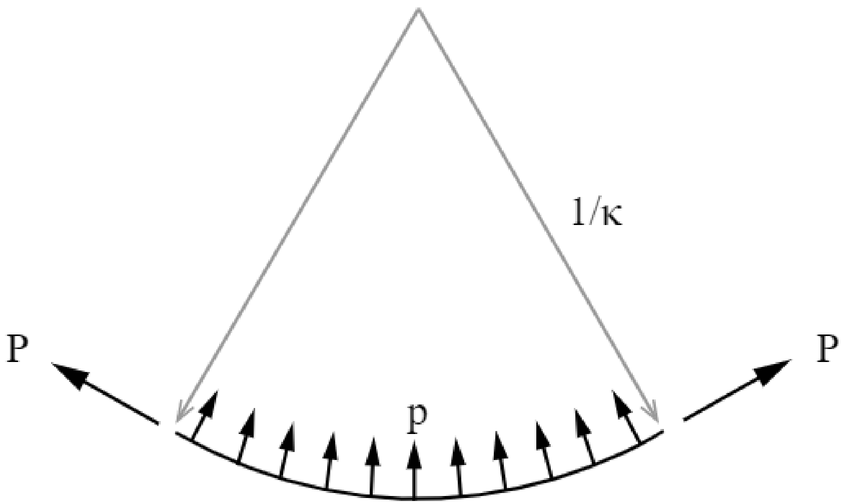
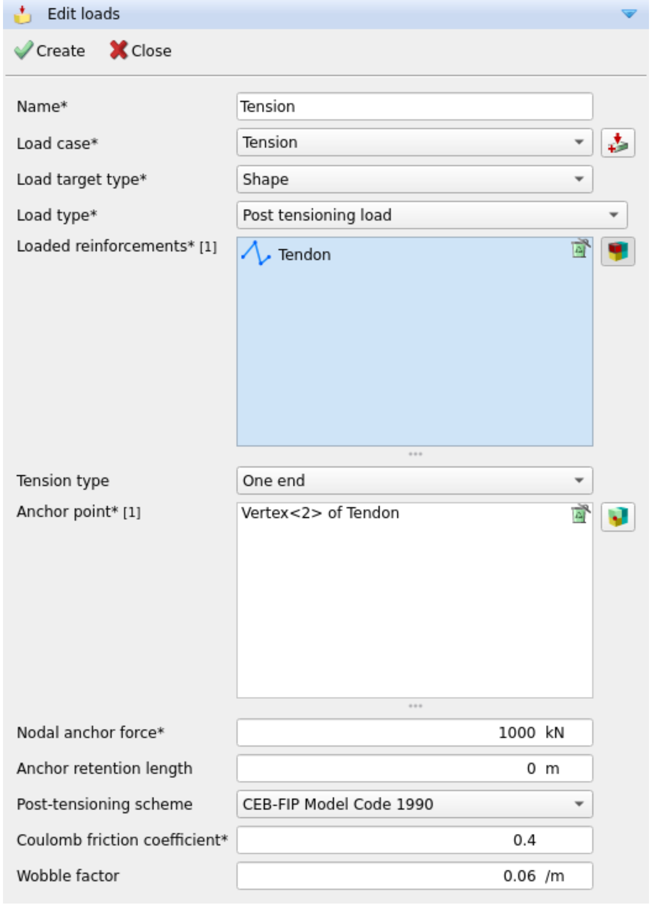


Figure 10: Pressure by curved tendon

We edit the defined post-tensioning load and specify a value of 0.4 for *Coulomb friction coefficient* and 0.06 /m for the *Wobble parameter* [Fig. 11] and click on 'Create'. Next, we rerun the analysis and check results.

DianaIE

Geometry browser → Loads → Tension → Edit loads [Fig. 11]
Main menu → Analysis → Run analysis
Results browser → Output → Reinforcements results → Reinforcement Cross-section Forces → Nx [Fig. 12]



Name*	Tension
Load case*	Tension
Load target type*	Shape
Load type*	Post tensioning load
Loaded reinforcements* [1]	Tendon
Tension type	One end
Anchor point* [1]	Vertex<2> of Tendon
Nodal anchor force*	1000 kN
Anchor retention length	0 m
Post-tensioning scheme	CEB-FIP Model Code 1990
Coulomb friction coefficient*	0.4
Wobble factor	0.06 /m

Figure 11: Post-tensioning load with friction loss

A reduction of force along the length of the tendon is observed in Figure 12.

The reduction is bi-linear with the slope changing from one parabola to another. We can verify this by calculating the force in the tendon at two points $X = 19$ m and $X = 20$ m from the anchor point, using the analytical formula. For parabola $g(X)$ Eq.(1) the reference point is the anchor point, $X = 0$. Thus P_0 is 1000 kN. The curvature κ is constant along the curve and is given by the second derivative of $g(X)$ with respect to x which equals to $g''(X) = 2 \times 0.006894 = 0.013788$ /m. Using this force at $\Delta r = 19$ m results in approximately 570 kN.

For parabola $h(X)$ Eq.(2) the reference point is $X = 19$ m since corresponding to end of the previous parabola. Thus P_0 is 570 kN, as calculated previously. The constant curvature κ is evaluated by taking the second derivative of $h(X)$ with respect to x as, $h''(X) = 2 \times 0.1 = 0.2$ /m. Using this force at $\Delta r = 1$ m, i.e $X = 20$ m results in approximately 515 kN.

Both calculated values are in line with the observed result [Fig. 12].

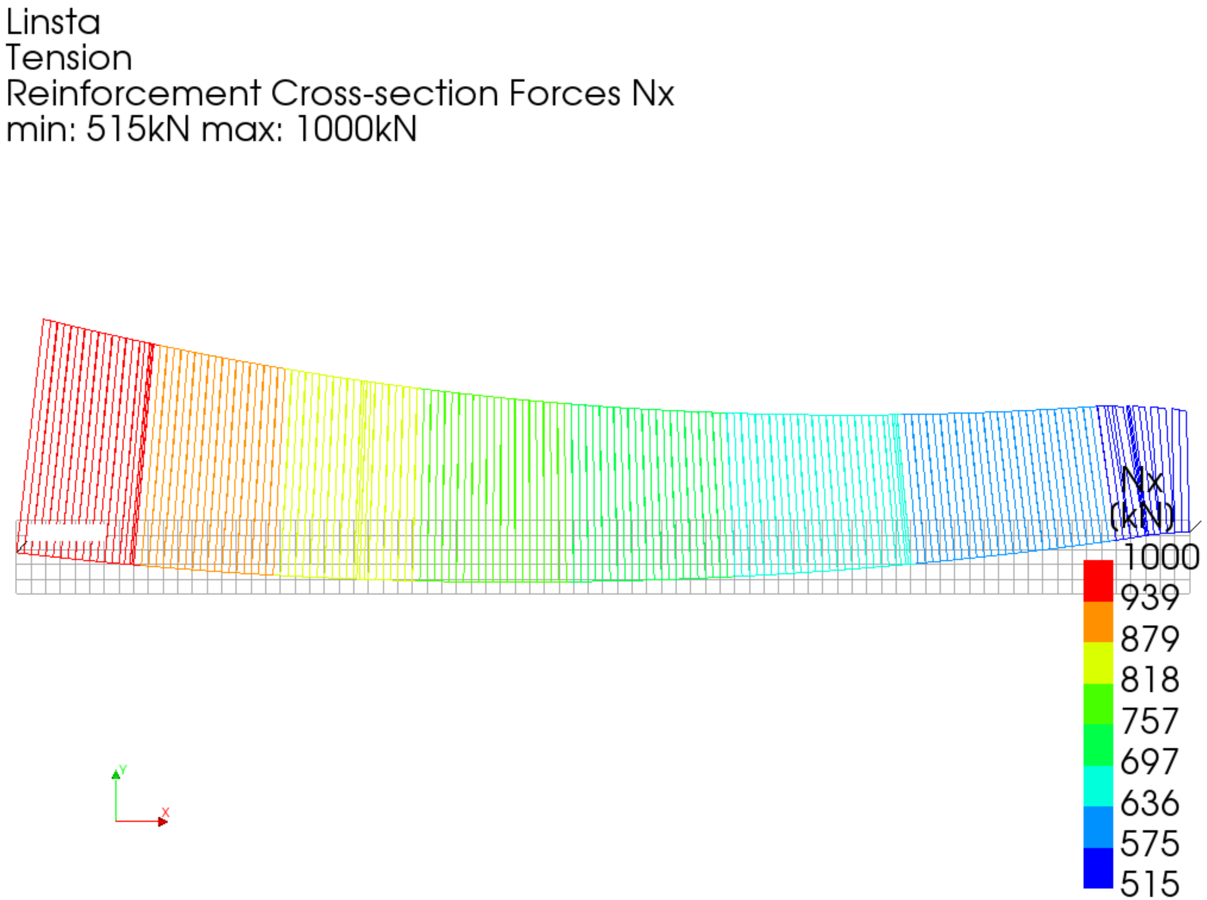


Figure 12: Cross-section force along the tendon

3.2.2 Anchorage

If the principle of anchoring is (partially or totally) based on the occurrence of friction forces between stressing elements and anchorage, then the stress is transmitted over a certain length of influence starting at the anchoring point [Fig. 13].

This stress causes a penetration of the anchorage ΔL which is called *retention length*. For a symmetrically distributed prestress reduction before and after penetration as seen in Figure 12, DIANAIE calculates the influence length Δx. This is done by searching for a point from the anchor until the following condition is satisfied:

$$\int_L \Delta P(x)dr >= \Delta L * E * A \tag{4}$$

where,

E is the Young's modulus of the tendon

A is the cross-sectional area of the tendon

If Δx becomes equal to the length of the tendon, DIANAIE reduces the prestress uniformly along the tendon until the condition Eq.(4) is satisfied.

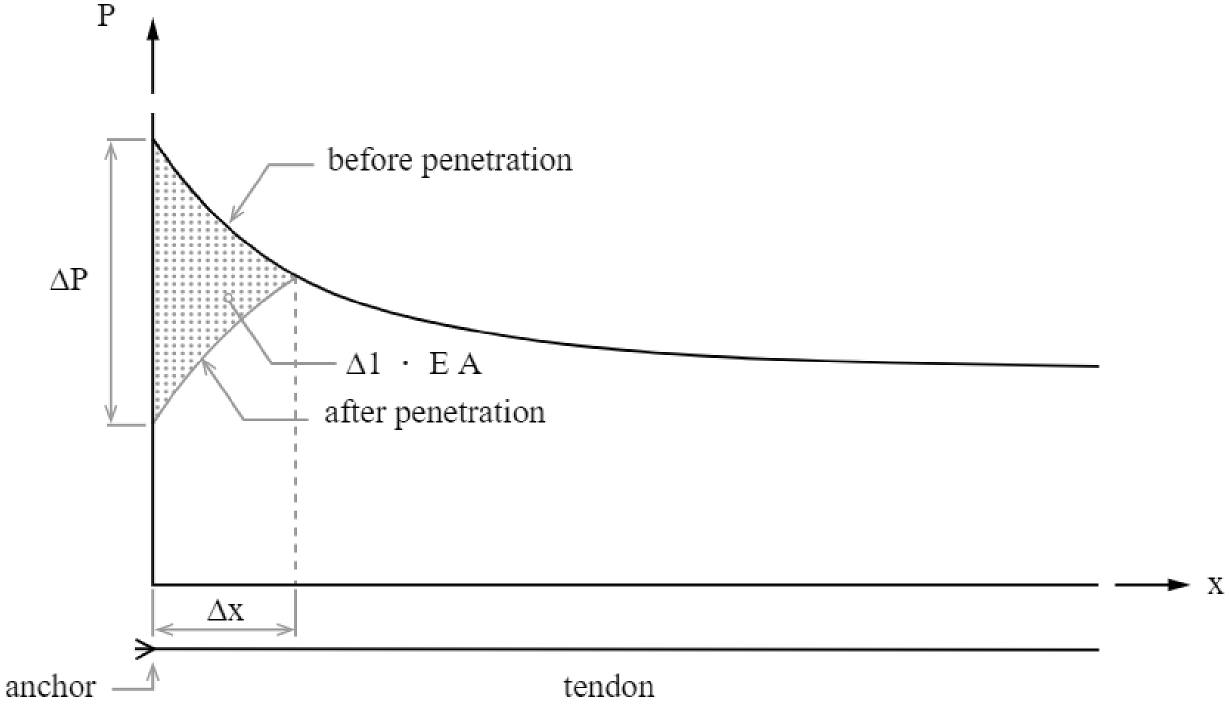
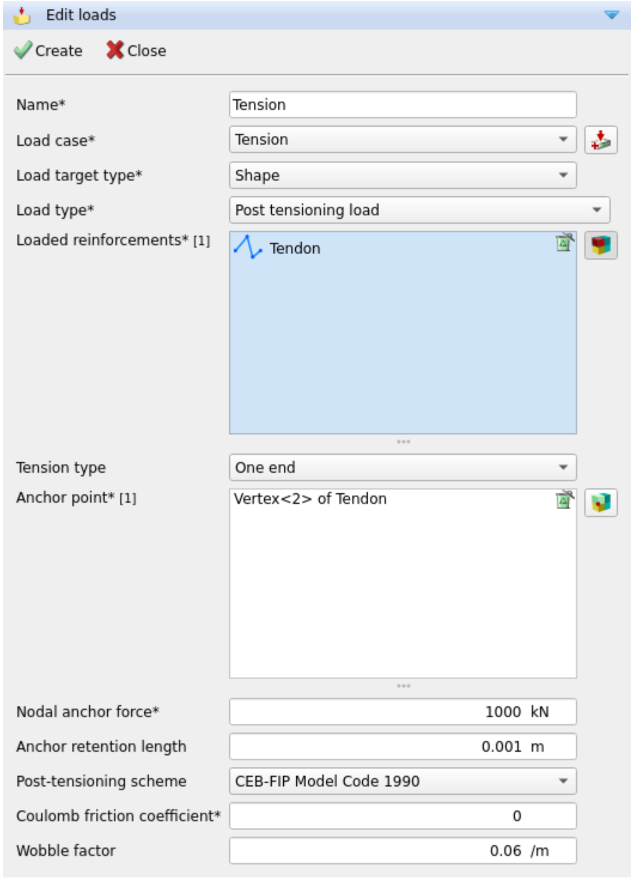


Figure 13: Loss of prestress in anchored tendon

In the model we consider an *Anchor retention length* of 0.001 m (1 mm) [Fig. 14]. We set the *Coulomb friction coefficient* to zero in order to see the effect of loss of anchorage without any friction losses along the tendon. Further we rerun the analysis and check results.

DianaIE

Geometry browser → Loads → Tension → Edit loads [Fig. 14]
Main menu → Analysis → Run analysis
Results browser → Output → Reinforcements results → Reinforcement Cross-section Forces → Nx [Fig. 15]



Name*	Tension
Load case*	Tension
Load target type*	Shape
Load type*	Post tensioning load
Loaded reinforcements* [1]	Tendon
Tension type	One end
Anchor point* [1]	Vertex<2> of Tendon
Nodal anchor force*	1000 kN
Anchor retention length	0.001 m
Post-tensioning scheme	CEB-FIP Model Code 1990
Coulomb friction coefficient*	0
Wobble factor	0.06 /m

Figure 14: Post-tensioning load with anchorage loss input

Since the *Coulomb friction coefficient* is set to zero, the pretress force along the tendon before the penetration of the anchorage follows the distribution seen in Figure 15. As the force is constant along the length, the condition to calculate Δx can only be satisfied by reducing the prestress uniformly along the length of the tendon. Thus Δx will be considered equal to the length of the tendon. The anchorage loss is therefore calculated as

$$\Delta P = (\Delta L * E * A)/L = (0.001 * 2 \times 10^8 * \frac{\pi}{4} \times 0.032^2)/20 = 8\,kN$$

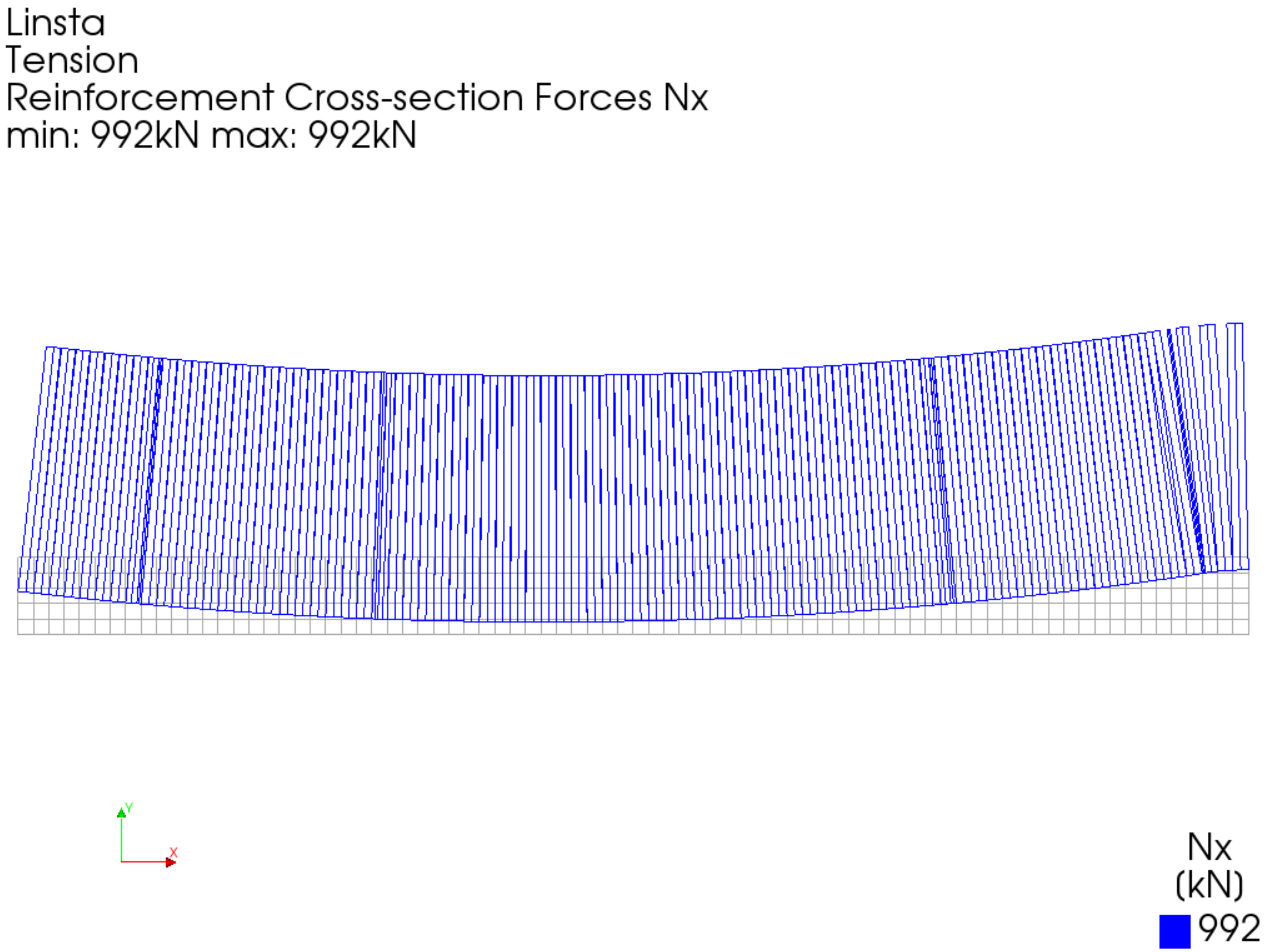


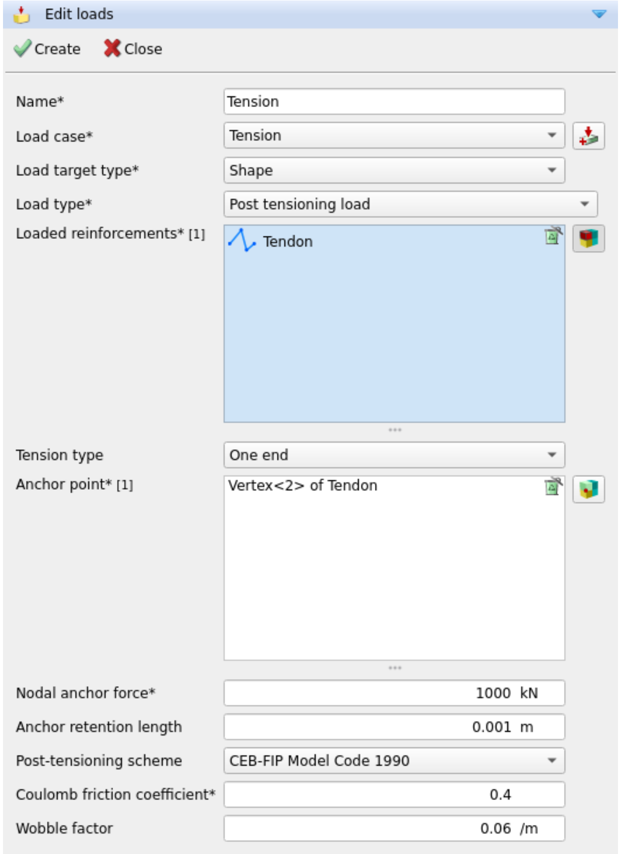
Figure 15: Force along tendon

3.2.3 Friction and Anchorage

We now edit the post-tensioning load to consider both anchorage and friction losses [Fig. 16]. Then we rerun the analysis and check results.

DIANAIE

Geometry browser → Loads → Tension → Edit loads [Fig. 16]
Main menu → Analysis → Run analysis
Results browser → Output → Reinforcements results → Reinforcement Cross-section Forces → Nx [Fig. 16]



Name*: Tension
Load case*: Tension
Load target type*: Shape
Load type*: Post tensioning load
Loaded reinforcements* [1]: Tendon
Tension type: One end
Anchor point* [1]: Vertex<2> of Tendon
Nodal anchor force*: 1000 kN
Anchor retention length: 0.001 m
Post-tensioning scheme: CEB-FIP Model Code 1990
Coulomb friction coefficient*: 0.4
Wobble factor: 0.06 /m

Figure 16: Post-tensioning load with friction loss input

As seen in the Figure 17, a force of 867 kN is observed at the anchor point. The force increases over a certain influence length untill a maximum of 934 kN and further decreases as we move leftwards.

The pretress force along the tendon before the penetration of the anchorage follows the distribution seen in Figure 12. As observed in the previous section, this distribution is linear. Therefore, the shaded region seen in Figure 13 is a triangle. The condition of anchorage loss can then be expressed as

$$0.5 \times \Delta P \times \Delta\times = \Delta L \times E \times A$$

The prestress loss at the anchor ΔP [Fig. 17] is 1000 - 867 = 133 kN. Thus the influence length can be calculated as

$$\Delta x = (\Delta L \times E \times A)/(0.5 \times \Delta P) = (0.001 \times 2 \times 10^8 \times \frac{\pi}{4} \times 0.032^2)/(0.5 * 133) \approx 2.4\, m$$

from the anchoring end, which is where the maximum value of force in the tendon is observed.

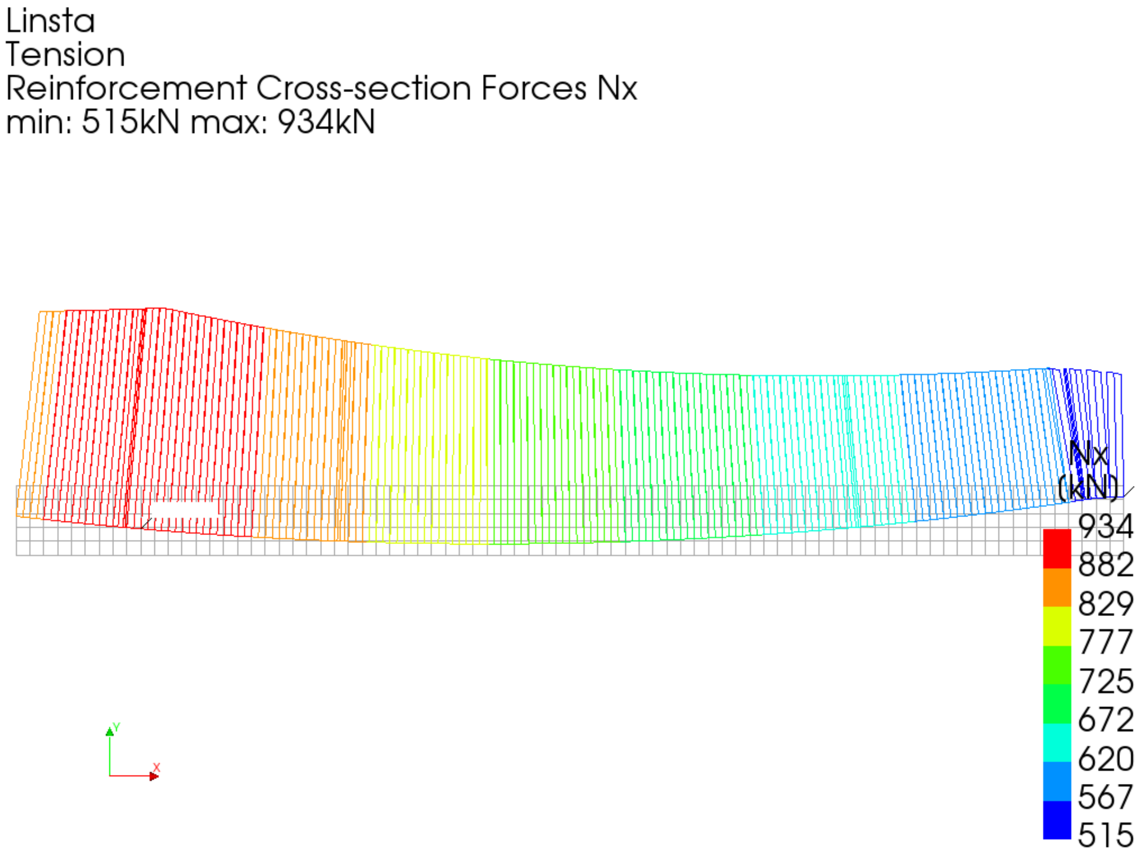


Figure 17: Force along tendon

Appendix A Additional Information

Folder: Tutorials/PostTensionLoad

Number of elements ≈ 450

Keywords:
- ANALYS: linear static.
- CONSTR: suppor.
- ELEMEN: bar cq16m pstres reinfo.
- LOAD: anchor postte reinfo.
- MATERI: elasti isotro nobond.
- OPTION: direct units.
- POST: binary ndiana.
- PRE: dianai.
- RESULT: cauchy displa extern force green reacti strain stress total.

DIANA FEA BV
Thijsseweg 11
2629JA Delft
The Netherlands
T +31 (0) 88 34262 00

DIANA FEA BV
Vlamoven 34
6826 TN Arnhem
The Netherlands
T +31 (0) 88 34262 00

WWW.DIANAFEA.COM

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