Frequency Response Analysis of a Generic Dam-Fluid-Foundation Model

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Abstract: The objective of this paper is to perform frequency response analysis of a generic dam resting on a foundation. The foundation is considered to be both rigid and flexible. The analysis is done for both dry dam and full reservoir conditions. A set of four distinct cases has been studied which are respectively as follows: 1) dry dam on rigid foundation, 2) dry dam on flexible foundation, 3) full reservoir on rigid foundation and 4) full reservoir on flexible foundation. The dam is assumed to be made up of concrete material. The fluid, when considered, is meshed with fluid elements and fluid-dam interaction is considered. The foundation flexibility is represented in the DIANA model by introducing interface elements with suitable values of stiffness. The interface elements are also assigned damping properties to represent the effect of the far-field. The bed of the fluid domain is assumed to be rigid and hence fully reflective. A preliminary eigenvalue analysis is done to determine the eigen modes of the system considered and subsequently a direct frequency response analysis is performed for all cases. The DIANA results are compared with the results for a similar model studied by Fenves and Chopra (1984). The results are shown in terms of accelerations in frequency domain. The DIANA results seem to be in good agreement with the results reported by Fenves and Chopra (1984).

Keywords: Fluid-Dam-Foundation Interaction, Frequency Response Analysis, Bottom Absorption, Damping, Sommerfeld Radiation, Flexible Foundation

Introduction

The dynamic analysis of dam-fluid-foundation interaction is of vital importance in engineering practice. Westergaard developed analytical expressions to compute the hydrodynamic pressure on a dam with vertical upstream face subjected to seismic ground motions. Later, with the advent of finite element analysis as an efficient numerical tool and with big leaps in computational abilities by computers, numerical analysis of dams have become easier and faster. In this paper, a dam resting on a rock foundation is analysed with and without reservoir. The main objectives are to show the capabilities of DIANA to consider the effects of fluid compressibility and foundation flexibility and compare DIANA results with those reported by Fenves and Chopra (1984).

Basic Assumptions and Definitions

The fluid elements in a model in DIANA require input of some basic material parameters like sonic wave speed to consider compressibility effects of fluid and conductivity (dummy) to generate the element conductivity matrix. The fluid boundary element used in DIANA model may represent three different boundary conditions – bottom absorption, free surface or a radiation (Sommerfeld) boundary. For radiation boundary elements, no element gravity convection matrix will be set up.

Background Theory

Figure 1 shows a generic fluid-structure interaction model. The fluid is characterized by a single dynamic pressure variable *p* and the coupling with the structure is achieved by consideration of interface forces and a standard finite element idealization. Assuming linear state of the fluid, the corresponding wave equation may be written as follows

$$\nabla^2 p = \frac{1}{c_s^2}\ddot{p} \qquad (1)$$

In the above equation, c_s is the wave speed defined by

$$c_s = \sqrt{\frac{\beta}{\rho}} \qquad (2)$$

The terms, β and ρ denote respectively the bulk modulus and the density of the fluid.

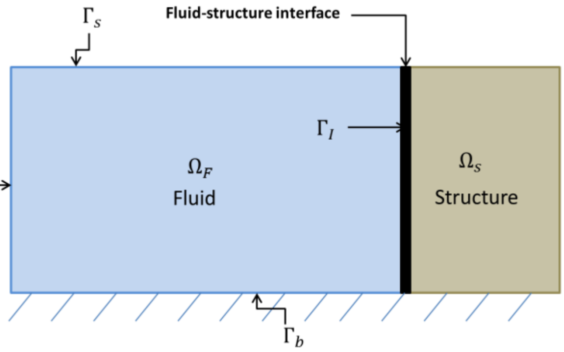

Figure 1 Fluid-structure interaction model

The fluid-structure interface obeys the following boundary condition.

$$\frac{\partial p}{\partial n} = -\rho_F \mathbf{n}_F^T \ddot{u}_F \; ; \; \sigma \mathbf{n}_s = p\mathbf{n}_F \qquad \text{on } \Gamma_I \qquad (3)$$

where, $\boldsymbol{n_F}$ and $\boldsymbol{n_S}$ are respectively the outward normal to the fluid domain and the outward normal to the structural domain. The continuity between the fluid and structural elements is ensured by keeping normal displacements of the fluid and the structure the same and following the condition as

$$\frac{\partial p}{\partial n} = -\rho_{\mathrm{F}} \mathbf{n}_F^T \ddot{u}_{\mathrm{s}} \qquad (4)$$

The free surface (at water surface) condition is given by the following equation.

$$p = \rho_{\mathrm{F}} g u_z \qquad \text{on } \Gamma_{\mathrm{s}} \qquad (5)$$

In the above equation, ***g*** is the acceleration due to gravity and ***z*** is the direction normal to the free surface. Using the following relation between the acceleration (normal to free surface) and change of pressure with respect to liquid depth,

$$\ddot{u}_z = -\frac{1}{\rho_{\mathrm{F}}} \frac{\partial p}{\partial z} \qquad (6)$$

one may get the linearized free surface condition for the first order waves by rewriting Equation (5) as

$$\frac{\partial p}{\partial z} = -\frac{1}{g} \ddot{p} \qquad \text{on } \Gamma_{\mathrm{s}} \qquad (7)$$

Sommerfeld's condition physically means that the waves produced and radiated from the sources must scatter to infinity with the velocity c_s and no waves may be radiated from infinity into the singularities of the field. This condition has been widely used to investigate incompressible water-wave radiation problems. This radiation boundary condition for fluid domain of infinite extent may be established as follows. It may be assumed that only plane waves would exist had the fluid boundary been placed at sufficiently large distance. In that case incoming waves are not present and the following equation becomes valid.

$$p = f(x - c_s t) \qquad (8)$$

where, +*x* denotes the outward direction. On taking partial derivatives of *p* with respect to space (*x*) and time (*t*) and thereafter eliminating *f* from the resulting equations, one may obtain the radiation boundary condition as follows.

$$\frac{\partial p}{\partial x} = -\frac{1}{c_s} \dot{p} \qquad \text{on } \Gamma_{\mathrm{e}} \qquad (9)$$

The above condition is termed as Sommerfeld radiation condition and would be applied in a plane normal to the direction of the wave speed. The term, c_s denotes the sonic wave speed of the fluid.

Another important aspect is to define a condition which is applicable at the bottom of the fluid domain. The reservoir bed partially reflects the waves and the extent of reflection or absorption depends upon the material of the bed which is represented by a wave reflection coefficient and is given as follows.

$$\frac{\partial p}{\partial n} = -\frac{1-\alpha_\beta}{c_s(1+\alpha_\beta)} \dot{p} \qquad \text{on } \Gamma_{\mathrm{b}} \qquad (10)$$

In the above equation, α_β is the wave reflection coefficient at the bottom of the reservoir. This coefficient is a ratio of the amplitude of the reflected hydrodynamic pressure wave to the amplitude of the propagating pressure wave incident on the reservoir bottom. The wave reflection coefficient, in DIANA, may vary between -1 (soft reservoir bottom material) to +1 (rigid reservoir bottom material). It may be mentioned here that Fenves and Chopra (1984) used a different formulation in their approach and have given an expression of wave reflection coefficient the value of which may vary between 0 and 1 (1 is used for rigid bottom).

Model Description

A generic two dimensional (2D) dam-reservoir-foundation model is developed. The model is shown in Figure 1. It consists of a triangular dam with height 100 m and base width 80 m. The dam contains water on one side and is underlain by a soil or foundation layer. The same model has also been studied earlier by Fenves and Chopra (1984). Though the assumptions and methodology of numerical modelling of the dam-reservoir-foundation system are different, an attempt has been made to simulate in DIANA the same model studied by Fenves and Chopra as closely as possible to obtain similar results. When the foundation is assumed to be rigid the foundation nodes are all supported through horizontal and vertical restraints as shown in Figure 2. The foundation flexibility is modelled with interface elements with suitable values of normal and tangential spring stiffness coefficients. The flexible foundation-dam-reservoir model is shown in Figure 3. The base nodes of the interface elements are fixed against translations to represent the fixity at the foundation base. The fluid-structure interaction is modelled by fluid-structure interface elements when the fluid is considered in the analysis.

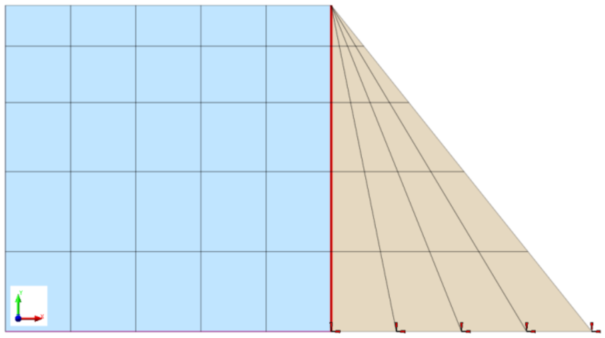

Figure 2 Model of reservoir on rigid foundation

The bottom absorption coefficient is used to simulate the elasticity of the fluid bed in respect of wave absorption phenomenon at the reservoir bottom. The fluid itself is modelled using fluid elements which have typically only dynamic pressure degree of freedom (and not displacement degree of freedom). The free surface of the fluid are assigned zero dynamic pressure to represent 'no surface wave' condition. The infinite extent of the fluid domain is modelled using suitable length of the fluid domain and using Sommerfeld radiation condition on the vertical face. It has been observed through some initial studies that the reservoir length 100 m is sufficient because any further increase in the extent does not give rise to any significant change in the responses.

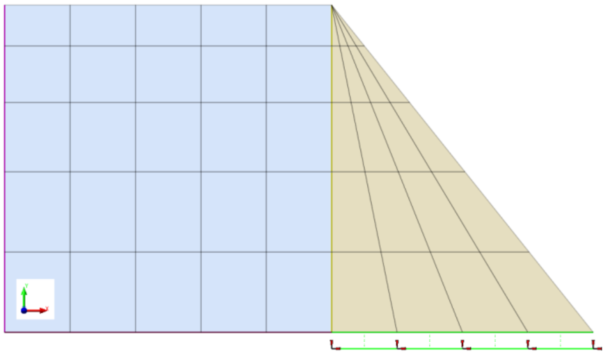

Figure 3 Model of reservoir on flexible foundation

Material Input Parameters and Loading

The material properties of dam, fluid and foundation are given in Table 1. The dam and the foundation are assigned a hysteretic damping of 10%. The damping coefficients assigned to interface elements for shear and compression wave absorptions, C_s and C_p respectively are calculated based on the following expressions.

$$C_s = \sqrt{\rho G} \tag{11}$$

$$C_p = \sqrt{\rho(\lambda + 2G)} \tag{12}$$

In the above two equations, G and ρ denote the shear modulus and the density of the medium. The parameter, λ depends on the elastic modulus, E and Poisson's ratio, ν of the medium. The expressions for G and λ are given below.

$$G = \frac{E}{2(1+\lambda)} \tag{13}$$

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \tag{14}$$

The damping coefficients, C_s and C_p are calculated to be 5.23 X 10^6 N-m/s^3 and 1.04 X 10^7 N-m/s^3 respectively. The type of interface elements used to represent foundation-soil interaction is L8IF. The properties of the interface elements are defined through damping coefficients as discussed above to simulate wave absorption phenomenon and the stiffness coefficients to simulate foundation flexibility. The values of the shear and normal stiffness of interface elements are determined as follows. The depth of the foundation is assumed to be the same as the height of the dam which is 100 m. The elastic modulus of the foundation is also assumed to be the same as that of the dam (2.7586 X 10^{10} N/m^2). Thus, on dividing the elastic modulus of the foundation by the depth of the foundation, the value of the normal stiffness modulus (K_n) of the interface elements is obtained as 2.7586 X 10^8 N/m^3. The tangential stiffness modulus (K_t) of the interface elements is usually the same or less than this value. As an initial guess, a pair of K_n= 2.7586 X 10^8 N/m^3 and K_t= 2.7586 X 10^8 N/m^3 has been chosen. With some more variations, it has been observed that K_n= 5 X 10^8 N/m^3 and K_t= 3.75 X 10^8 N/m^3 yielded results which are close to the results obtained by Fenves and Chopra (1984) for the same model. So, these values are used when foundation flexibility is considered. The values of material parameters are given in Table 1.

Parameter & Unit	Dam	Fluid (water)
Elastic Modulus (N/m^2)	2.7586X10^{10}	-
Poisson's ratio	0.20	-
Density (kg/m^3)	2482.00	1000.00
Hysteretic damping ratio	0.10	-
Sonic wave speed (m/s)	-	1438.66
Conductivity	-	1.0

Table 1 Values of material parameters

A unit horizontal acceleration for all excitation frequencies is applied at the base (at supported nodes) of the model.

Case Studies

A set of 4 different cases has been studied – Case 1) Rigid foundation without water, Case 2) Flexible foundation without water, Case 3) Rigid foundation with water and Case 4) Flexible foundation with water. The water is considered to be compressible in both cases. So, the first two cases represent dry dam and the next two cases represent full reservoir (filled up to its height). In case 3, zero dynamic pressure heads are used at the surface of the water as well as on its lateral boundary. In case 4, the zero dynamic pressure head is applied at the surface and the lateral boundary is assigned Sommerfeld radiation condition and this is done to simulate the same case as reported by Fenves and Chopra (1984) even more closely. The model for Case 4 is shown in Figure 4. The eigen analysis of the model corresponding to Case 1 is performed to understand the mode shapes and obtain the free vibration frequencies. The direct frequency response analysis is carried for each of these cases. The results from direct frequency analysis are presented in terms of absolute horizontal acceleration response of the dam crest. A comparative study is made for various values of base absorption coefficients when water is considered in the model (i.e. Case 3 and Case 4). It is worth mentioning here that for these 4 cases the value of the wave reflection coefficient (α_β) is kept at 1.0 assuming the fluid bed to be very rigid.

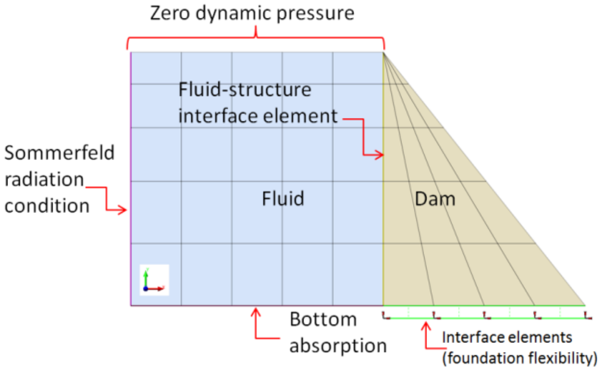

Figure 4 DIANA model for dam on flexible foundation with compressible fluid

Results and Conclusions

The first eigen frequency, thus obtained from an eigen value analysis, is used to normalise the frequency axis of the response spectrum. Figure 5 shows the 1st and the 2nd dominant mode shapes of the model representing Case 1. Both these modes are active in horizontal direction. The first dominant eigen frequency is around 4.60 Hz. This value is used to normalize the excitation frequencies to present the frequency response curves. The third dominant mode shape is in the vertical direction with a frequency of 11.90 Hz. Figures 6 and 7 demonstrate the frequency spectra of absolute horizontal acceleration response at the crest of the dam for first two cases, i.e. empty dam on rigid and flexible foundations respectively. The DIANA results are compared to the results for same cases as reported by Fenves and Chopra (1984). The matching seems to be very close.

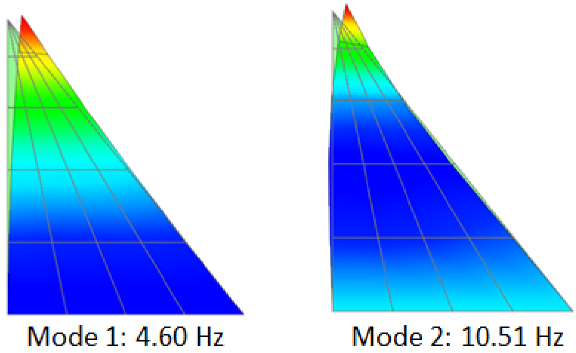

Figure 5 1st and 2nd mode shapes for dry dam on rigid foundation

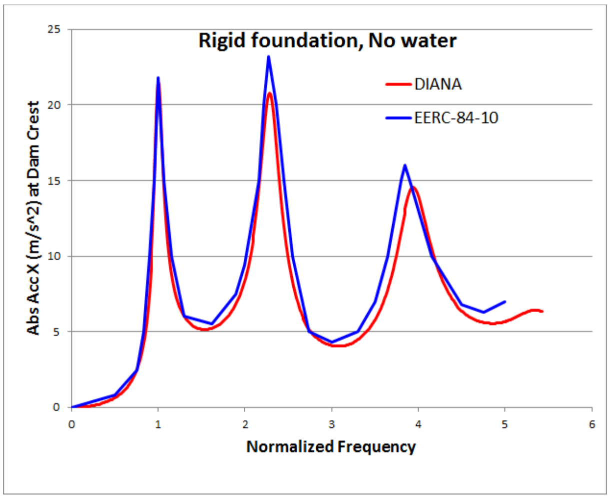

Figure 6 Acceleration response spectra (Case 1)

Figures 8 and 9 depict the frequency response spectra in terms of absolute values of horizontal acceleration at the dam crest when foundation flexibility is considered for Case 3 and Case 4 respectively. It is worth mentioning here that for the model in Case 4 (corresponding to Figure 9) in which both compressibility of water and flexibility of foundation are taken into account, it is observed from the results that the flexible modes of fluid vibration in conjunction with the flexible foundation modes are producing several amplified peaks in the response curves. This may be seen in Figure 9 for the case with 'DIANA – Hysteretic Damping'. In order to get rid of these dominant peaks the hysteretic damping (10%) in the dam is now replaced by Rayleigh damping in DIANA model and the corresponding results may be seen in Figure 9.

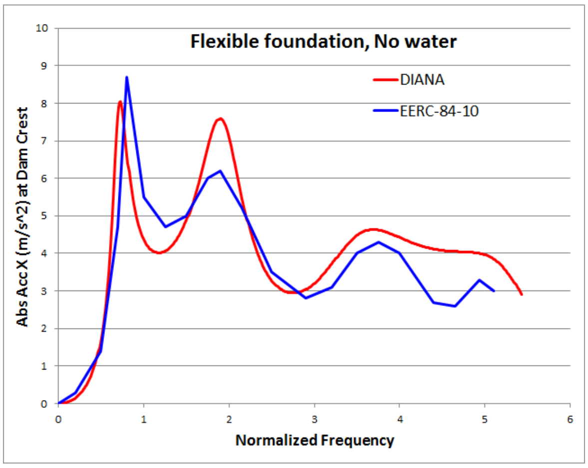

Figure 7 Acceleration response spectra (Case 2)

The Rayleigh damping parameters are calculated according to Equation (15) on the basis of minimum and maximum cut-off frequencies of 1 Hz and 6 Hz respectively (corresponding to normalized frequencies of 0.22 and 1.30 on the frequency spectrum). This includes only the first dominant mode pertaining to the dam vibration (around normalized frequency = 0.6, Figure 9) and thus avoids contribution from higher modes of fluid vibration. A constant damping ratio of 10% is considered. This frequency range is chosen to obtain DIANA results closer to results reported in Fenves and Chopra (1984).

$$2\zeta\omega = \alpha + \beta\omega^2 \qquad (15)$$

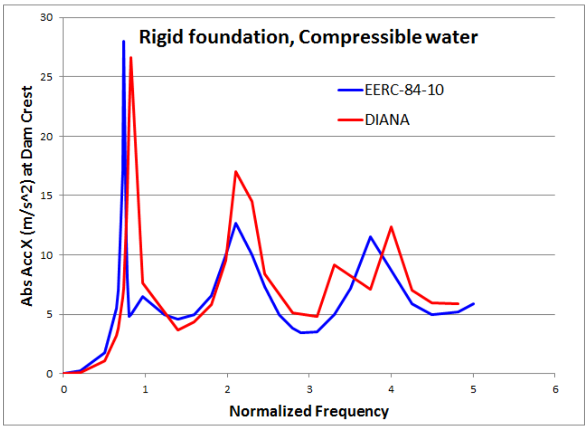

Figure 8 Acceleration response spectra (Case 3)

It may be of interest to readers to see the results

for Case 2 (dry dam on flexible foundation) when the model is working without damping in interface elements. Figure 10 shows the result for such a case in terms of absolute horizontal acceleration response at dam crest in direction. This figure shows the difference in results due to the presence and the absence of damping properties in the interface elements. It is observed that without interface element damping the response at the dam crest is significantly amplified due to nearly undamped rocking of the dam.

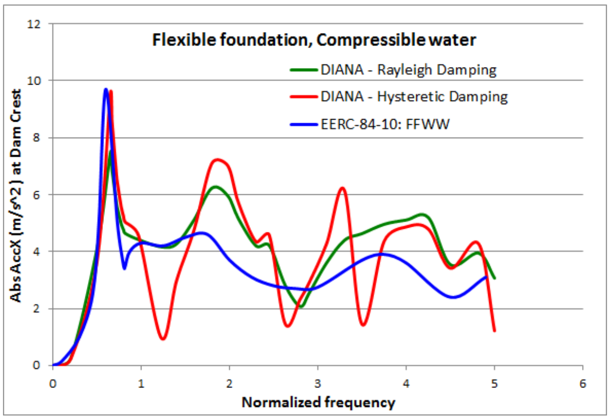

Figure 9 Acceleration response spectra (Case 4)

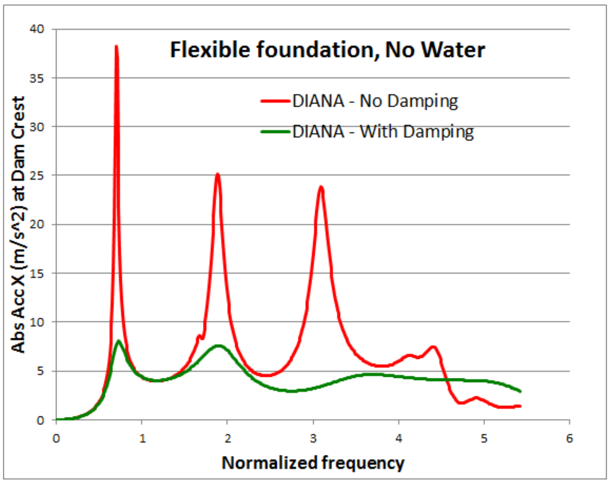

Figure 10 Acceleration response spectra of a dry dam on flexible foundation (Case 2) with and without damping properties in interface elements

Thus, despite the differences in assumptions and approach, the case studies on the generic dam model with and without reservoir generated results in DIANA which seem to be in close agreement with the results reported by Fenves and Chopra (1984).

References

Gregory Fenves, Anil K Chopra. “Earthquake Response Analysis and Response of Concrete Gravity Dams”, Report Nr. UCB/EERC-84/10, August 1984.