Strength Reduction Analysis

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Abstract: This paper explains the basic theory, concept and application of strength reduction method relevant to geotechnical engineering practices. Some simple models on slope stability are developed in DIANA in two-dimensional and three-dimensional spaces and the results are compared with the standard analytical results and with results obtained from other numerical methods or software. The failure patterns in each model are also captured and shown in separate figures.

Introduction

The analysis of slope stability problems in geotechnical engineering is of paramount importance. It is often needed to predict the degree of safety associated with the slope and the tentative failure pattern of the slope. The strength reduction method is typically used for the assessment of slope stability where dominantly a Mohr-Coulomb or similar material model is used. This will allow the users to specify only a certain part of the model to be subjected to strength reduction instead of the whole model. In DIANA, the strength reduction analysis must be preceded by a standard nonlinear static analysis.

Basic Assumptions and Definitions

Unlike limit equilibrium method, no assumptions need to be made about the location or shape of the failure surface or lateral forces on the sides of the slices and their directions. The critical failure mechanism in a complicated model may assume any shape. The factor of safety in case of slope stability analysis by strength reduction method may be defined as the ratio of the resisting shear strength of the material to the driving shear stress developed along the failure plane.

Background Theory

In strength reduction method the strength characteristics of the soil materials are reduced by a factor until the loss of stability or failure of the structure occurs. The reciprocal of this reduction factor is identified as the factor of safety associated with the soil model under investigation. In DIANA strength reduction method is implemented as a separate module, named REDUCT. Thus, cohesion ***c*** and friction angle ϕ are reduced to assess the slope stability. The main output of this type of analysis is the factor of safety.

First the self-weight and any additional loads are applied on the structure using a standard phased nonlinear static analysis and an equilibrium state is obtained. Next the module REDUCT is run to determine the factor of safety iteratively. A nonlinear analysis is carried out in each iteration. The iterative procedure to determine factor of safety is discussed below.

The iteration starts with the factor of safety, $\boldsymbol{FS_n = FS_0}$, where $\boldsymbol{FS_0 = 1}$ unless otherwise specified by the user. Subsequently, $\boldsymbol{FS_n}$ is incremented by $\Delta\boldsymbol{FS}$ to $\boldsymbol{FS_{n+1}}$ ($\boldsymbol{FS_{n+1}} = \boldsymbol{FS_n} + \Delta\boldsymbol{FS}$) where $\Delta\boldsymbol{FS}$ is a user input. With the updated ***FS***, the cohesion ***c*** and the tangent of the friction angle (i.e. tanϕ) are scaled down as follows.

$$c_{n+1} = \frac{c}{(FS)_{n+1}} \tag{1}$$

$$tan\ \phi_{n+1} = \frac{tan\phi}{(FS)_{n+1}} \tag{2}$$

The resulting yield surface is shown in Figure 1 in which ***c*** and ϕ are scaled down to $\boldsymbol{c_f}$ and ϕ_f respectively.

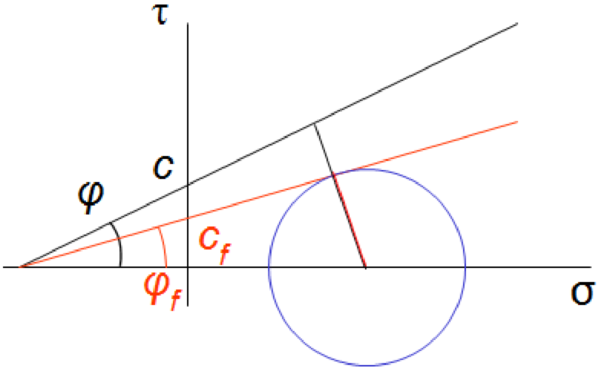

Figure 1 Mohr-Coulomb yield surface after strength reduction

Now with the reduced strength parameters a new equilibrium is sought by carrying out a nonlinear static analysis. If a new equilibrium is found, i.e. the analysis converges, then ***FS*** is increased and a new equilibrium is sought again with reduced strength parameters. The process is repeated until the analysis diverges or does not converge within a certain number of iterations, e.g. 50 or user-defined failure criteria, e.g. maximum equivalent plastic strain is exceeded. At this stage the increment in the factory of safety (Δ***FS***) is set to half of its previous value. This decreases the factor of safety ***FS*** to a lower value so that equilibrium can be found again. This iterative procedure continues until Δ***FS*** becomes smaller than a user specified tolerance, so that ***FS*** is determined with sufficient accuracy.

Now, the nonlinear static analysis that is carried out in each iteration to find the equilibrium with reduced strength parameters is discussed. First,

the reduced stress due to reduction of the strength parameters is computed. In order to do that, the elastic strain corresponding to the stress σ_0 from the preceding nonlinear analysis is calculated as

$$\varepsilon_0 = [SE]^{-1}\sigma_0 \qquad (3)$$

In the above equation, **[SE]** is the elastic stress-strain matrix. Once $\boldsymbol{\varepsilon_0}$ is available the reduced stress σ is obtained by invoking the Mohr-Coulomb (or Mohr-Coulomb like) material routine with $\boldsymbol{\varepsilon_0}$ as the total strain. Finally the out-of-balance force, i.e. the residual force, ***g*** to find the equilibrium is computed as

$$g = \int B^T\,(\sigma_0 - \sigma) \qquad (4)$$

In Equation (4) the term *B* denotes the strain-displacement matrix and the integration sign indicates integration over element level leading to element force vector followed by an assembly process to obtain the global force vector. The residual force *g* is applied to the model with reduced material properties and displacements at equilibrium are calculated.

In DIANA, a new material parameter STRRED is introduced to be used in table of the input file along with the other material parameters of Mohr-Coulomb or Drucker-Prager model. The presence of this keyword in a particular material model (*strred cohphi*) indicates that this material and the corresponding mesh set to which it will be assigned may undergo reduction in strength parameters (i.e. cohesion and tangent of friction angle). Presently only Mohr-Coulomb and Drucker-Prager models are considered in DIANA. Future extension may include Modified Mohr-Coulomb, Hoek-Brown, and Coulomb friction (for interface elements) models. The following section shows two examples of slope stability analyses which are solved using strength (c-phi) reduction method as discussed above in this section.

Example 1 – 2D slope stability analysis

The first example consists of a two-dimensional (2D) slope as shown in Figure 2 below. The 'Y' direction is the vertical and 'X' is the horizontal one. The corresponding material properties are shown in Table 1. The model geometry is meshed with high order plane strain element. The base of the model is constrained against translations in both directions (X and Y) and the left vertical side of the model is fixed against horizontal (X) motions.

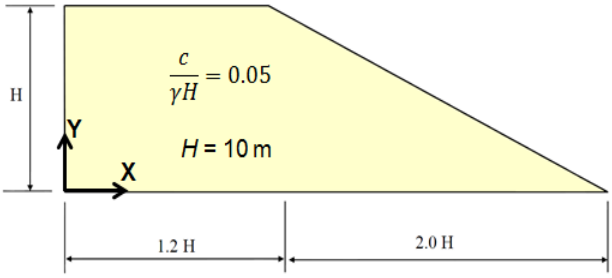

Figure 2 2D model for slope stability analysis

Parameter	Value	Unit
Elastic modulus	10^5	kN/m^2
Poisson's ratio	0.3	--
Unit weight	20	kN/m^3
Cohesion	10	kN/m^2
Friction angle	20	^{0}
Dilatancy angle	0	^{0}

Table 1 Material parameters for the soil model used in 2D slope stability analysis

The factor of safety for the slope considered in the above example is found to be around 1.33. This value is reasonably close to the value of factor of safety obtained from Bishop's method (1.38). The deformed mesh and the equivalent plastic strains developed in the model at the end of the analysis are shown in Figures 3 and 4 respectively. Figure 3 shows the maximum deformations (failure line) in red colour and Figure 4 depicts the total strains developing along the slope of the failure in the model.

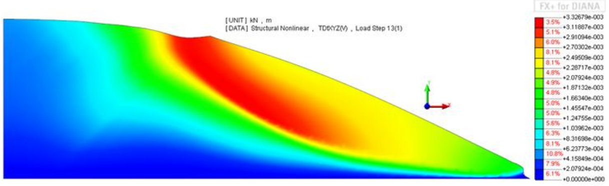

Figure 3 Deformed 2D model at failure

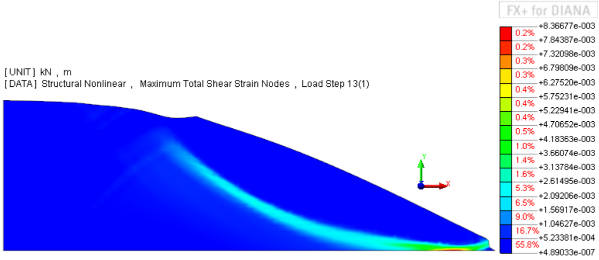

Figure 4 Maximum total shear strains in the 2D model

Example 2 – 3D slope stability analysis

This example demonstrates the failure of a 3D slope which has the geometry as shown in Figure 5. The corresponding material properties are given in Table 2.

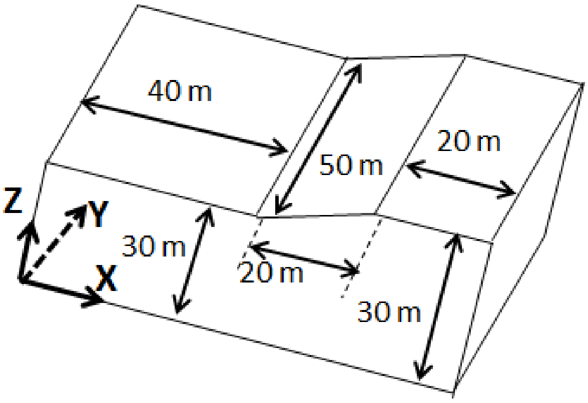

Figure 5 Geometry of 3D slope stability model

Parameter	Value	Unit
Elastic modulus	10^6	kN/m^2
Poisson's ratio	0.3	--
Unit weight	17	kN/m^3
Cohesion	15	kN/m^2
Friction angle	20	^{0}
Dilatancy angle	0	^{0}

Table 2 Material properties for the soil model used in 3D slope stability analysis

The lower boundary of the model is fixed and the lateral sides on XZ plane and YZ plane are restrained against movements in Y and X directions respectively (Z being the global vertical axis). The value of the factor of safety obtained as a result of the strength reduction analysis is about 1.72. The same model when analysed with SAS-MCT4.0, UTEXAS3, STABL5M and PLAXIS3D generated the values of factor safety shown in Table 3 below.

	Strength Reduction Analysis				
Tool used	SAS-MCT 4.0	UTEXAS3	STABL5M	Plaxis3D	DIANA
FOS	1.70	1.70	1.71	1.80	1.72

Table 3 Values of factor of safety of the 3D model obtained from different analysis tools

The factor of safety for this 3D slope agrees well with most of these values from other pieces of software. The deformation and total strains developed are shown in Figures 6 and 7 respectively.

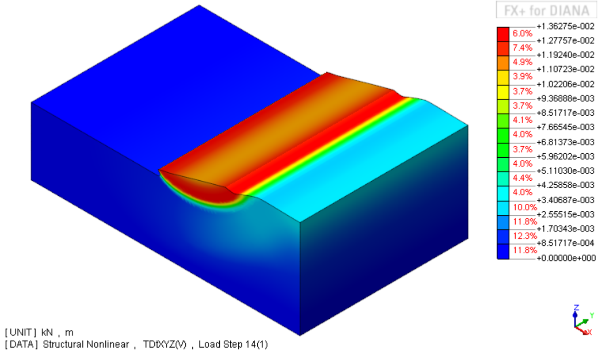

Figure 5 Deformed 3D model at failure

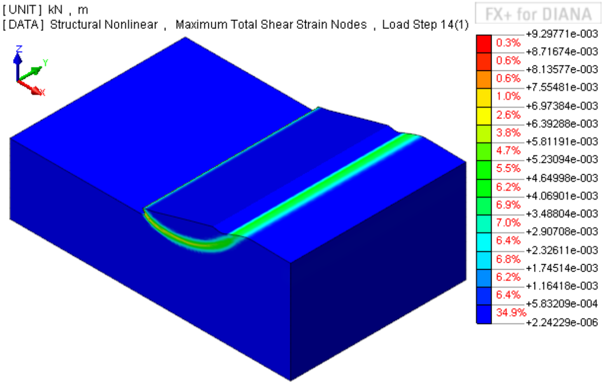

Figure 6 Maximum total shear strains in the 3D model